PERCEIVED SIZE AND DISTANCE IN VISUAL SPACE

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BY ALBERTA S. GILINSKY

In a recent survey of the status of research on visual perception, Graham (8) points out that the facts of space discrimination are still poorly understood. "In particular a long series of researches has not resulted in those systematically determined functional relations which, as Holway and Boring (1941) say, are 'wanted and wanting'" (8, p. 876).

The purpose of this study is to develop a quantitative formulation of visual space perception expressing such functional relations. Functions relating visually perceived size and distance to true size and distance are rationally derived and then applied to the results of a variety of visual size and distance experiments.

The theory is expressed quantitatively in two simple, interrelated formulas—for perceived distance and perceived size, respectively. These two formulas are derived mathematically in three different ways: First, the two formulas are rigorously derived from the basic metric of visual space as established mathematically (for binocular vision) by Luneburg (12). Second, the same two formulas are mathematically derived (somewhat less rigorously but without restriction to binocular vision) from the known principles of visual perspective. Finally, the same two formulas are derived by a simple inductive method of mathematical composition for the two boundary laws of size constancy and retinal image (visual angle). All three methods of derivation yield the identical pair of formulas to express a unifying law of visual space perception.

In place of three different observer constants and several unstated scale constants contained or required to be supplied in Luneburg's formulas, the present simplified formulas contain essentially a single constant or parameter, A. This parameter, A, varies for different observers and with the conditions of the experiment. The numerical value of A for a given observer may be expected to depend upon the availability of cues to distance. Through the perfection of techniques whereby the value of A may be reliably determined for individual observers and for different conditions, the two formulas can be made into a useful tool to enable individuals to estimate true sizes and distances from visual observations at a distance. A practical measure or specification may also thus be made available for classifying or grading individuals with respect to their distance judgment or spatial discrimination.

The general validity of the two formulas is supported by visual size and distance experiments planned and conducted for this study. They are also checked by visual size experiments by prior investigators (9, 16) and other observations recorded in the literature, such as perceived size in stereoscopic photographs (18), the alley experiments (2), and the visual speed of moving objects (5). A further striking check (seemingly unpredicted) is the applicability of the same distance-formula to perceived distance determined by *auditory* stimulation (17). This variety of evidence indicates that the derived relations and the unifying law which they express may have considerable generality.

Visual Space. We open our eyes

and perceive a three-dimensional world in which objects have perceived size, form, and localization. Our experience is called *visual space*.

Visual space and physical space are not identical. Neither is visual space in three dimensions a *proportional* replica of objective space in three dimensions. One is a distorted transformation of the other. As Ogle has stated:

". . . an object in objective space may be displaced without deformation and without changes between related points, . . . in visual space, this is not true. As an object recedes, not only does it appear to become smaller but its shape appears to change, this change being greater in the depth extent than in height or width. Many other discrepancies between the two spaces have been established. . . . It is only important that, whatever relationship exists between objective and visual space, it should be fairly stable if the individual is to act effectively in the physical world" (13, p. 11).

In the prior search for a simple law of visual space perception, two principal theories have been held—each incompatible with the other, and each beyond certain limits becoming inconsistent with known phenomena and relations.

1. At comparatively short distances, well within the range of accustomed experience or confirmation by other senses, the organism tends to utilize corrective cues so as to compensate for the distance and the corresponding diminution of retinal size. Observations under such limiting conditions have given rise to the so-called law of size constancy, whereby the perceived size of an object is taken as constant and independent of retinal size (1, 8, 9).

2. At the other extreme, if size of retinal image is assumed to be the sole cue to perceived size, and if all depth or distance perception is ignored, the so-called law of the retinal image or visual angle is obtained, whereby the perceived size of an object is taken as simply proportional to the size of the retinal image (or visual angle) and therefore inversely proportional to the distance of the object. This theory is more nearly true at very great or astronomical distances (1, 3, 8, 9).

Neither the law of the retinal image nor the opposite theory, the law of size constancy, can be sustained over the entire range of distances and conditions (2, 4, 8, 9). As Boring (2, 4) has shown, each view describes only a special limiting case. An adequate general theory must be capable of satisfying these boundary conditions and of providing, as well, for a smooth continuous transition over the large intervening range between the extreme boundary laws. The aim is to find a unifying law applicable to all distances and to all conditions of visual perception.

The various theories of perceived size may also be compared by relating them to another concept—"the projection theory of visual perception" (10, 12). If the two base points are the centers of rotation of the eyes and the projection lines are the optical axes, the intersection of the two projection lines will be at the actual point fixated and therefore at the true physical distance D. If the retinal image is projected to this true physical distance D, the perceived size will be undiminished by distance and the law of size constancy is obtained. If, on the other hand, the retinal image is always projected to an arbitrary constant distance, independent of true distance, the perceived size will be simply proportional to the size of the retinal image, without regard to distance, and the law of the retinal image is obtained. As stated above, neither of these two extreme laws is adequate.

The perception of distance or depth is a fact, and must be included in a generalized theory. But that does not mean the true or physical distance D, for *physical* distance is not perceived. What is sensed or perceived is the *perceived distance* d, and that is the only distance that is subjectively realized and therefore the only distance at which the projection can, logically and consistently, be made. If the size (in radians) of the retinal image or visual angle is φ (phi), and if the perceived distance is d, then the *perceived size* s, is given by the simple relation

$$s = \varphi \cdot d. \qquad (1)$$

According to our hypothesis, this is the basic, unifying relation between perceived size and perceived distance.

The significance of this concept for our understanding of space perception is that it unifies a large number of previous observations, embracing both size constancy and retinal image as limiting cases, and bridging the gap between these two extremes.

Perceived distances are foreshortened. The perceived distance d increases with the true distance D but at a reduced and diminishing rate. Consequently the law of perceived size given by equation (1) is intermediate between the law of retinal image and the law of size constancy.

Moreover, the perceived distance d approaches a finite limit A when the true distance D increases to infinity.

As Luneburg has stated: "Astronomical objects like the sun or the moon are seen at finite distances; their sensed size is also finite and in no way proportional to astronomical dimensions. Even the sky itself gives the impression of a dome of finite radius" (12, p. 1).

As thus pointed out by Luneburg, and earlier, by James (10), there exists a maximum limit of perceived distance. Whereas physical space is infinite, visual space is finite. This concept of an upper limiting threshold (or limen) for spatial discrimination or distance perception is consistent with the known facts of spatial discrimination and with sensory discrimination in general (8, 10). The question as to what physiological mechanism or receptor process governs or underlies this maximum limit of visual distance is immaterial for our purpose.

Luneburg was principally concerned with investigating the geometrical character (hyperbolic, Euclidian, or elliptic) of the tridimensional manifold comprising visual space. He shows that "the geometry in any manifold can be derived from its metric, i.e., from a rule for measuring small line elements." The problem is thus "to establish a metric for the manifold of visual sensations" (12, p. 2). The basic metric given by Luneburg establishes the general relation between the two spaces, visual space and physical space. Using Luneburg's basic metric as our point of departure, we shall derive our own psychometric coordination, based on the Euclidian geometry, for formulating the relation between visual and physical space.

Definitions: Perceived Size and Distance

The real *objective* size of a physical object is subjectively unknowable. We can, however, apply the familiar physical and geometrical operations of measurement, starting with the simple relation of equality, so as to determine the *relative* physical sizes of objects. A standard unit measuring rod supplies the objective scale. The objective size measured on this scale will be designated by S_0.

Similarly, the *subjective* size *s* of visual objects cannot be objectively determined. We can, however, apply various procedures of matching and comparison, limited to an individual observer and constituting psychophysical measurement. Starting with the simple relation of equality, we may thus determine (for a given observer) the *relative* perceived sizes of different objects, and also the relative perceived sizes of the same object at different distances. Moreover, we can select one of these viewing distances as "the normal viewing distance" δ (delta) and we can call the corresponding perceived size of the object the "true size" *S* to be used as a standard reference size for the visual perception of the object. This concept has been anticipated by William James:

"*Out of all the visual magnitudes of each known object we have selected one as the* REAL *one to think of, and degraded all the others to serve as its signs.* This 'real' magnitude is determined by aesthetic and practical interests. It is that which we get when the object is the distance most propitious for exact visual discrimination of its details. This is the distance at which we hold anything we are examining. Farther than this we see it too small, nearer too large. And the larger and the smaller feeling vanish in the act of suggesting this one, their more important *meaning*" (10, p. 179).

The physical measuring rod used for objective size becomes also the measuring rod for subjective size provided the rod is held at the normal viewing distance δ (delta). We assume equality of *visual* size when object and measuring rod have equal physical size and are held at identical viewing distance.

When the object is more remote, we hold the measuring rod at the normal viewing distance δ and then transfer its visual size to superposition upon the object at its perceived distance *d*. The equivalent procedure is to transfer the visual size of the distant object to the measuring rod (or comparison standard) held at the normal viewing distance δ. This technique must be included in the instructions to the observer and must be learned by practice. In each case a recalled visual size must be compared with an observed visual size. The two visual sizes (object and standard at disparate distances) must not be compared directly by simultaneous viewing because that tends to introduce comparison of retinal images instead of comparison of subjective sizes.

Finally, we have to define and establish a measure of *subjective distance*. For this purpose we note that the transformation from physical to visual space is cubic or isometric at short distances such as the normal viewing distance δ and that the depth dimension becomes perceptively compressed at greater distances. Accordingly, the same measuring stick used for perceived size may also be used as the measuring stick for perceived distance; in each case it should be held at the normal viewing distance δ.

A physical cube will be perceived as a cube if viewed at the normal viewing distance δ. At more remote distances the perceived thickness or depth will diminish more rapidly than the frontal dimensions. The receding solid will cease to appear as a cube in visual space unless it is physically elongated as it recedes to increasing distance from the observer. The sun and the moon lose their spherical thickness and appear as flattened disks against the sky.

A line in physical space may be bisected. A line in visual space may be bisected. The two division points are not necessarily identical. If the line recedes from the eye; as a distance, the two divisions will be different.

A yardstick in physical space may be moved to different points or positions without changing its *physical* length, but its visual length will change. A yardstick in visual space may be moved to different points or positions without changing its *visual* length, but its physical length would have to be changed. The constant physical yardstick is used for measuring physical lengths. The constant visual yardstick is used for measuring visual lengths (perceived sizes and distances).

In order to correlate the two yardsticks, we define the *visual* yardstick as the perceived size of the *physical* yardstick held at a convenient normal viewing distance δ (delta) from the eye. By this expedient we get around the difficulty of correlating two incommensurables—an objective magnitude and a subjective magnitude.

To apply the visual yardstick to a distant object, it is assumed that the visual yardstick is moved to the distant object without changing the *visual* length of the yardstick.

A distance from the observer (*O*) is measured physically by repeatedly applying the physical yardstick, thereby dividing the distance into *physically* equal divisions. The successive divisions will appear shorter as they recede from the observer. To measure the distance *visually*, the visual yardstick must be applied repeatedly so as to divide the distance into *equal visual divisions.* These successive divisions will be longer as they recede from the observer.

The meaning of perceived size s is thus linked to the meaning of perceived distance d. Visual spatial extents are measured in all three dimensions of visual space by one and the same subjective measuring rod or visual yardstick. (The physical length of this yardstick will vary with distance and direction. Only at the standard viewing distance δ are the physical and visual yardsticks equal and invariant.)

For the purpose of the present study, it is extremely important to draw a clear distinction between perceived distance and estimated distance, corresponding to the similarly important distinction drawn by Hering and others (2) between perceived size and estimated size. Perceived distance is phenomenal or apparent distance, comprising exclusively the direct product of stimulation (visual and muscular). Estimated distance includes an intellectual correction of perceived distance, derived from past experience and training, to arrive at a more informed inference or judgment of true distance. Perceived distance is represented by the reduced magnitude d. Estimated distance is an attempt, conscious or unconscious, to estimate (from d) the true distance D.

When a subject says that a distant object appears to be one mile away, he means that it appears as far away as an object known to be one mile away. That is an absolute judgment (based on past experience) and not a measure of perceived distance. For a consistent definition of *perceived distance* d, when we say that a perceived distance is 40 yards we mean that the distance is perceived (in a subjective scale of visual distance) as twice as big as a perceived distance of 20 yards, or 40 times as big as a perceived distance of one yard—40 *visual* yardsticks laid end to end. The physical lengths of these visual yardsticks would increase progressively as they recede from the eye. Accordingly, the perceived distance of an object 100 yards away may be only 30 or 40 yards, although we may have learned, by training or experience, to judge it as 100 yards away. Similarly, the perceived distance of the horizon or the moon may be only 50 yards away by this definition.

For a consistent definition of *perceived size s*, the constant visual yardstick must be applied (in imagination or projection) to the distant object at its *perceived* distance. *O* should be guided to concentrate on the apparent size of the distant object at the distance at which *O* projects or visualizes the distant object.

Imagine an arm, capable of indefinite extension, holding the visual yardstick, maintaining the apparent length of the stick unchanged as it moves through space to make possible a subjective superposition of the measuring stick upon the distant object, say, the moon. If the imaginary arm carries the measuring stick farther than the perceived distance of the moon, the resulting measurement of perceived size will be too big. If the distance to which the visual yardstick is carried falls short of the perceived distance of the moon, the resulting measurement of perceived size will be *too small* (cf. 11, p. 237). The various answers given by different individuals to the question, How large is the moon?—answers which vary from ½ inch to 30 feet—illustrate this strikingly. Instructions to concentrate on the apparent size of the moon at its perceived distance yield equivalence matchings within a much narrower range, 4 to 20 inches. The variation which remains must be largely attributed to the fact that individuals differ in their perception of spatial depth or distance, particularly when, as in this case, the actual distance is so great and beyond the range of experience through the senses.

PERCEPTION OF DEPTH

In the perception of visual space the two essential elements are size of *retinal image* (or visual angle) φ (phi) and the perception of depth or distance *d*. Both retinal and muscular sensations are involved.

In binocular vision the basic essential for perception of depth or distance is the *convergence* (or the horizontal disparity or parallax) γ (gamma). The interocular or interpupillary distance provides the psychophysical base line for the geometry of binocular space perception. Although Luneburg refers to γ sometimes as "horizontal disparity" and sometimes as "bipolar parallax," he writes:

"The meaning of γ is clear: the angle subtended by the lines of sight at the point of *convergence, P*" (12, p. 13).

In monocular vision the corresponding physiological cue is the muscular or kinaesthetic sensation of *accommodation*.

In addition to these primary or basic cues of convergence and accommodation, there is a variety of supplementary clues which aid the perception of spatial depth. These associated or auxiliary clues include linear perspective, aerial perspective, interposition, movement parallax, texture gradient, light and shade, and apperceptions from past experience.

Luneburg does not attempt a physiological explanation of the relation of size perception to convergence, but he does consider this relation to be a necessary hypothesis. "Our scale of size seems to contract with increasing values of γ" (12, p. 103).

We need not consider the specific physiological mechanism whereby depth or distance in visual space is perceived. Such discussion is not essential for the development of the main thesis of the present study.

THE BASIC FORMULAS

The mathematical derivations of the two fundamental interrelated formulas for perceived distance and perceived size are given in the Appendices. All three methods of derivation yield the identical pair of basic formulas.

The basic formula for *perceived distance* is:

$$\frac{d}{D} = \frac{A}{A + D}, \qquad \text{(I)}$$

where d = perceived distance, D = true (physical) distance, and A = maximum limit of perceived distance for a given O under given conditions.

In fact, the constant A is self-defined by the formula. For $D = 0$, $d = 0$; but for D = infinity, $d = A$. In other words, A is the apparent distance of objects at infinity. Perceived distance d increases with true (physical) distance D; but the curve of d plotted on D, starting with the slope $d/D = 1$, flattens out and becomes asymptotic to the limiting horizontal line, $d = A$.

This limiting value, $d = A$, in the basic formula is given by the apparent distance of astronomical objects; it is also equal to the visual distance from the observer to the perspective horizon (vanishing point for parallel lines receding from the observer).

The O constant or parameter A is a measure of the finite depth of visual space for a given O under given conditions. An O with a higher value of A has an expanded visual world with a wider or farther horizon. An O with a lower value of A has a more compressed or limited visual world with a closer sky and a closer horizon.

The fact that the sky generally appears like a flattened bowl indicates that A has a lower value for pure space perception (elevated vision without foreground) and a higher value for horizontal vision (with foreground), due to differences in clues and context (the "moon illusion").

The high relative compression of visual space into a limiting boundary compared with the corresponding infinite expanse of physical space is impressively illustrated by the following numercial example (calculated for a convenient assumed value of A = 100 ft.):

Physical Scale (ΔD)		Visual Scale (Δd)
The first	100 ft. of D will be perceived as 50	ft. = $\frac{1}{2}A$
The next	200 ft. of D will be perceived as 25	ft. = $\frac{1}{4}A$
The next	400 ft. of D will be perceived as 12.5	ft. = $\frac{1}{8}A$
The next	800 ft. of D will be perceived as 6.25	ft. = $\frac{1}{16}A$
The next	1600 ft. of D will be perceived as 3.125	ft. = $\frac{1}{32}A$

Summation (limit)

For D = infinity, d = 100 ft. = A.

Accordingly, distance discrimination ($\Delta d/\Delta D$) rapidly diminishes with distance.

The larger the value of A, the more nearly perceived distance d approximates true distance D. For A = infinity, perceived distance would be equal to true distance.

The plotted functions representing equation (I) for several values of A are shown as a family of curves in Fig. 1. Perceived distance d is plotted against true distance D, in the same units and to the same scale. The diagonal straight line forming the upper limiting boundary represents the function for true or undiminished distance perception. It is obtained from equation (I) with A = infinity. The curves below this boundary line express equation (I) with respective numerical values of A corresponding to 50, 60, 70, 80, 90, 100, 120, and 200 ft. Larger values of A shift the position of the function upward toward the diagonal line for true distance; smaller values of A shift the function downward toward minimum perception of distance, corresponding to the lack of adequate cues for distance perception.

The basic formula for *perceived size* is

$$\frac{s}{S} = \frac{B}{A + D}, \qquad \text{(II)}$$

where s = perceived size, S = subjective true size (perceived size at normal viewing distance δ,) and $B = A + \delta$. The terms A and D retain the definitions of equation (I). The correction term δ (delta) is the distance at which an object is viewed for subjective true size. This correction makes perceived size equal to true size ($s = S$) not at $D = 0$, but at some small value of $D = \delta$. Thus, an object appears to be true size when it is presented at some normal viewing distance (about 2 feet) in front of the observer. This constant δ may vary with the O, also with different objects for the same O. This device permits us to measure perceived size s (at various distances) and subjective true size S in the same units and to the same scale. The subjective scale by which s and S are measured is not involved and need not be known or expressed. We work only with the ratio s/S. Both are subjective magnitudes, and the scale is immaterial.

Accordingly, the three distance terms (A, B, D) may be given in one set of units, and the two size terms (s and S) in another set of units.

The plotted functions representing equation (II) are shown as a family of curves in Fig. 2. The ratio of perceived size to true size (s/S) is plotted as a function of real distance D. For A = infinity, the upper horizontal boundary line is obtained, representing $s/S = 1$; this relation is size constancy. For $A = 0$, the bottom hy-

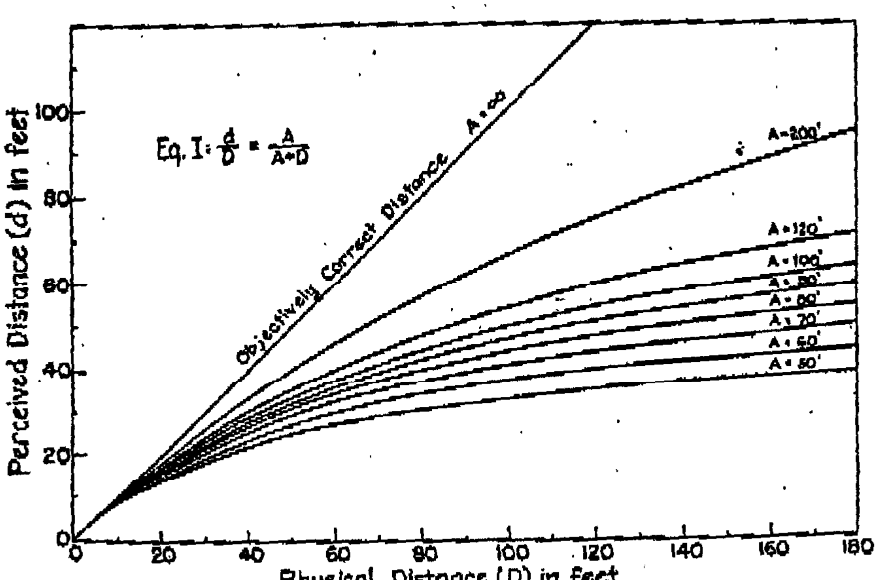

FIG. 1. Theoretical functions for perceived distance. The graphs of equation (I) showing perceived distance plotted against physical distance for various assumed values of A. For $A = \infty$ equation (I) yields the diagonal line representing veridical distance perception. Below this boundary line decreasing values of A displace the curves downward toward minimum distance perception.

perbolic boundary curve is obtained, representing $s/S = \delta/D$; this is the law of retinal image. Both of these special cases or boundary laws are given by equation (II) upon substituting the appropriate values of A (zero or infinity). Intermediate values of A (such as $A = 50$ to 200 feet) yield the intermediate curved graphs shown in Fig. 2.

The single parameter A in both basic formulas thus governs the position of the respective functions between the limiting boundary laws. For perceived distance, the magnitude of A determines the position of the visual distance function between the two extremes of $d = D$ (or perceived distance identical with true distance) and $d = 0$ (or greatly diminished distance perception). In similar manner, the magnitude of A governs the position of the visual size function between the two boundary conditions representing size constancy and the opposite extreme of perceived size proportional to and dependent only upon retinal image or visual angle.

Accordingly, the magnitude of A is a simple, convenient, and direct measure or index of what has been termed by prior investigators "phenomenal regression to the real object" (16). A zero value of A represents zero "regression," and an infinite value of A represents complete or one hundred per cent "regression." An advantage of such use of A over other indices of "regression" is that it is a constant for a given O under given conditions over the entire range of an experiment, whereas other indices vary from observation to observation or from point to point of a plotted experimental curve.

The empirical value of A (for a given O under given conditions) may be determined by any two or more

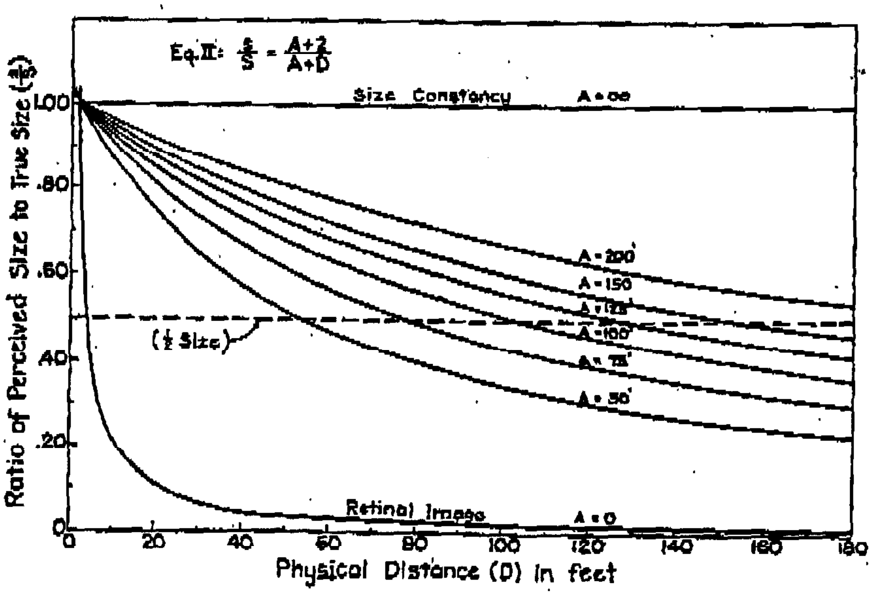

FIG. 2. Theoretical functions for perceived size with distance variant. The graphs of equation (II) for various assumed values of A. The perceived size is shown as ratios to the true size (perceived size of the stimulus object at a normal viewing distance, taken as 2 feet). The value of A governs the position of the functions between and inclusive of the horizontal solid line representing the law of size constancy and the bottom curve representing the law of the retinal image or visual angle. The horizontal dashed line cuts the curves at $s = \frac{1}{2}S$, indicating the distance at which this ratio may be expected for different values of A.

observations at different distances, or by a single observation of an astronomical object. (See Appendix IV.)

Instead of concentrating upon the effect of varying the parameter A, we may also study equation (II) with reference to the effect of varying the distance D. For $D = \delta$, $s = S$, and the object is seen as "true size." As the object is brought closer to the eye, perceived size s increases somewhat; and when the object is moved away from the eye, perceived size diminishes. (Luneburg's derived formulas yield the anomalous result of making perceived size diminish steeply when the object is brought close to the eye. See his Fig. 41.)

In further reference to equation (II), for small values of the distance D (relative to A), s/S is close to unity, corresponding to the law of size con-constancy; and for very large distances D (relative to A), s/S is greatly diminished and varies inversely with D, corresponding to the law of retinal image.

The basic formulas (I and II) are thus seen to express a law intermediate between size constancy and retinal image while yielding these two extreme conditions at short viewing distances and at astronomical distances, respectively.

In the following sections the two basic formulas (I and II) will be applied to recorded results of prior experiments and to new experiments involving observations of perceived size and distance. These data will provide an initial empirical test of the validity and practical usefulness of the present theoretical formulation.

EXPERIMENTAL CONFIRMATION

Perceived Size. A number of previously recorded experiments, in which the subject adjusts the size of a comparison stimulus at a constant distance to match a standard stimulus at a variable distance, yield results which correspond neither to the law of retinal image nor to the law of size constancy.

Instead, beginning with the first systematic experiment on perceived size (Martius, 1889), the size matches consistently follow an *intermediate law* (2, 8, 9, 11, 16). The dependence of perceived size upon distance is found to be in good agreement with the prediction expressed by equation (II) for perceived size. The slope of the function relating the adjusted size of the comparison object and the distance of the standard object is affected by controlling circumstances (4, 8, 9, 18). These variations are reflected by appropriate changes in the magnitude of the parameter A in equation (II).

The most adequate investigation of this problem was performed by Holway and Boring (9). Their experiment measured perceived size at distances up to 120 feet, and also showed how the resulting functions shifted with the reduction of the available cues to distance.

O's task was to match a controlled standard disk at distances varying from 10 feet to 120 feet. The size of the standard disk was made proportional to the distance so that it always subtended an angle of one degree, thus presenting constant retinal image or visual angle. The varying apparent size of this controlled standard disk was measured by means of a variable comparison disk which remained always at 10 feet from *O*.

The results for two sets of conditions, binocular regard and direct monocular regard, for Boring himself as *O*, are shown in Fig. 3. Binocular vision gives a function close to the ascending straight-line function corresponding to size constancy. Reduction to monocular vision lowered the function nearer to the horizontal slope corresponding to perceived size dependent only upon retinal image. Both of the curves drawn through the data points are given by equation (II) for perceived size.

For the upper curve, obtained for binocular vision, the parameter A is 243 feet; for the lower curve, obtained for monocular vision, the parameter A is 132 feet. The value of A is reduced when the perceptive cues are reduced. For both equations, B is conveniently taken as equal to A plus 10 feet, 10 feet being the distance of the comparison object. The same formula (equation II) applies to both binocular and monocular vision, with only a change in the parameter A.

From these results we learn that the value of A is, in fact, governed by the available cues to distance. The value of A determines the position of the intermediate function (expressed by equation II) between the two extremes, size constancy and retinal image. The constant A is thus demonstrated to be a sensitive index or measure of "phenomenal regression" and of the conditions which affect its attainment.

Perceived Distance by Equal-Apparent Intervals. In connection with this study two pilot experiments were performed, dealing with the relation of perceived distance d to physical distance D. Both experiments were designed to serve as an initial test of equation (I) for perceived distance, and as a guide to more refined experimentation.

Experiment I used a modified method of equal appearing intervals. O stood at one end of an unfamiliar indoor archery range, about 80 feet

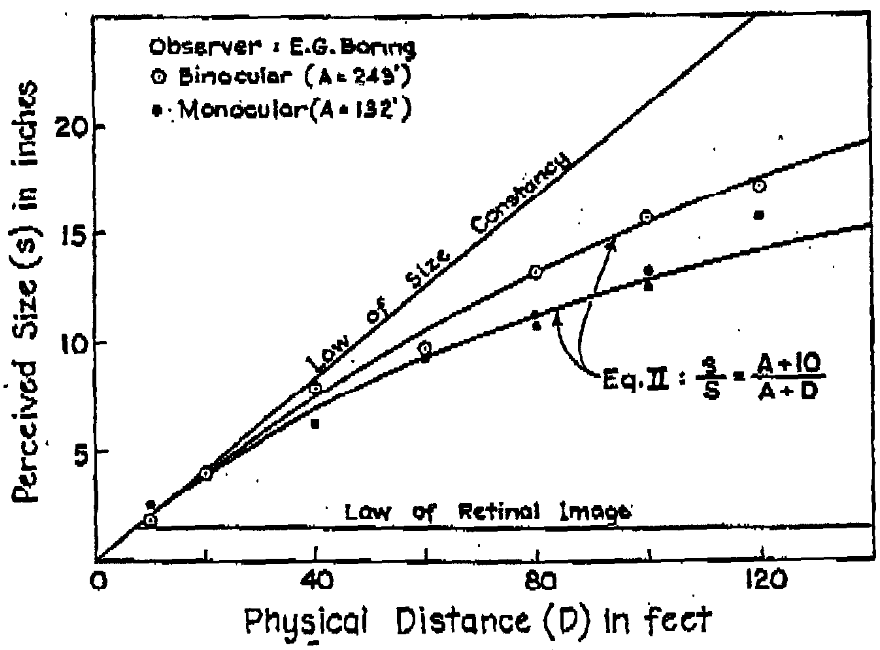

FIG. 3. Perceived size as a function of distance for binocular and monocular observation. The data are from Holway and Boring (9) with their own correction for space error. Equation (II) fits both sets of observations with only a change in the parameter A. As the conditions are altered from direct monocular regard to direct binocular regard, the parameter A has a higher value, shifting the function upward to approach the top diagonal line for size constancy. This line rises because they increased the size of the standard in proportion to the distance to keep visual angle constant.

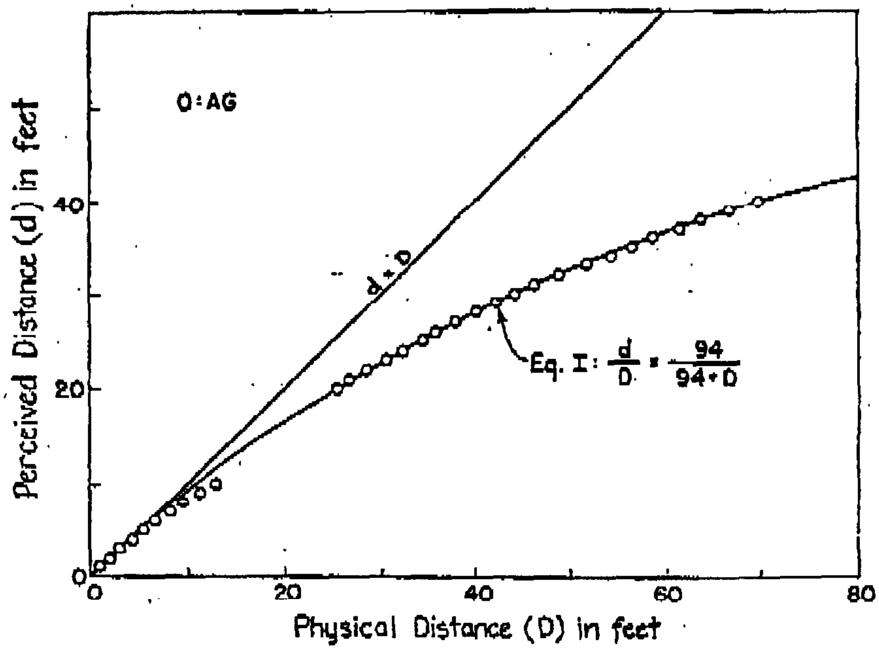

FIG. 4. Perceived distance as a function of physical distance (Exp. I). The circles represent successive equal apparent increments of distance from 0 to 10 subjective feet, and from 20 to 40 subjective feet. At 10 subjective feet the O was told to indicate the distance that appeared twice as great, thus determining the physical magnitude to be assigned to 20 subjective feet. The data lie on the curve of equation (I) with $A = 94$ feet. The diagonal line represents veridical distance perception.

ong, and directed the experimenter, who moved a pointer stick at a slow and nearly constant rate along the ground away from O, to mark off successive increments of equal perceived length. Each successive increment of perceived distance was an attempt to match, by non-simultaneous viewing, a memorized "subjective foot rule" in the case of one observer and a memorized "subjective meter stick" in the other. Successive division points were temporarily marked by a horizontal rod marker which was moved as soon as each next step had been attained.

The results for the two Os are shown in Figs. 4 and 5. In order to illustrate the manner in which the data are analyzed, Table I presents for one observer the cumulative measured distance D in physical units for each value of subjective distance d; the calculated value of A for each separate observation ($A = D\,d/(D-d)$; Appendix IV, equation 5); and the theoretical value of D from equation (I), using the weighted mean value of A determined from these data. It is significant to note the experimental constancy of A over the series of observations, particularly toward the larger values of D; also the correspondence between the measured and the calculated values of D. (For smaller distances, small experimental deviations are reflected by larger variations in the calculated value of A.)

In Figs. 4 and 5, perceived distance d is plotted against physical distance D in the same units and to the same scale. The diagonal straight line is the theoretical function for perceived distance identical with true measured distance. The obtained points lie on the tangent curves given by equation (I). The agreement between obtained and theoretical values is striking.

Perceived Size of Astronomical Objects. An independent determination of the constant A was made for the two Os of Experiment I by means of direct binocular observation of the perceived size of an astronomical object, the full moon.

Using the method of constant stimuli in two categories, the observer was presented with a series of cardboard disks ranging from 4 inches to 30 inches in diameter. The O was instructed to hold each disk at normal viewing distance and, by regarding it alternately with the moon (which was slightly above the horizon), to judge the disk as larger or smaller than the moon.

Ten judgments by the O were obtained for each disk within the transition zone, and the stimulus which received an equal number of "larger" and "smaller than" judgments was determined for each observer. The results of this procedure are as follows:

For O "J. B." the apparent size of the moon is $s = 9.4$ inches $= 0.78$ ft.; and by Appendix IV, equation (2), $A = 87$ ft.

For O "A. G." the apparent size of the moon is $s = 11$ inches $= 0.92$ ft., and hence $A = 102$ ft.

The values of A thus determined are in reasonably close agreement with the values of A (about 94 ft.) obtained from Experiment I on perceived distance.

Perceived Distances by Fractionation. In the second of our pilot experiments, Experiment II, the method of fractionation (bisection) was employed in order to construct a ratio scale of visual distance and to determine the relation of visual distance to the stimulus magnitude over a greater

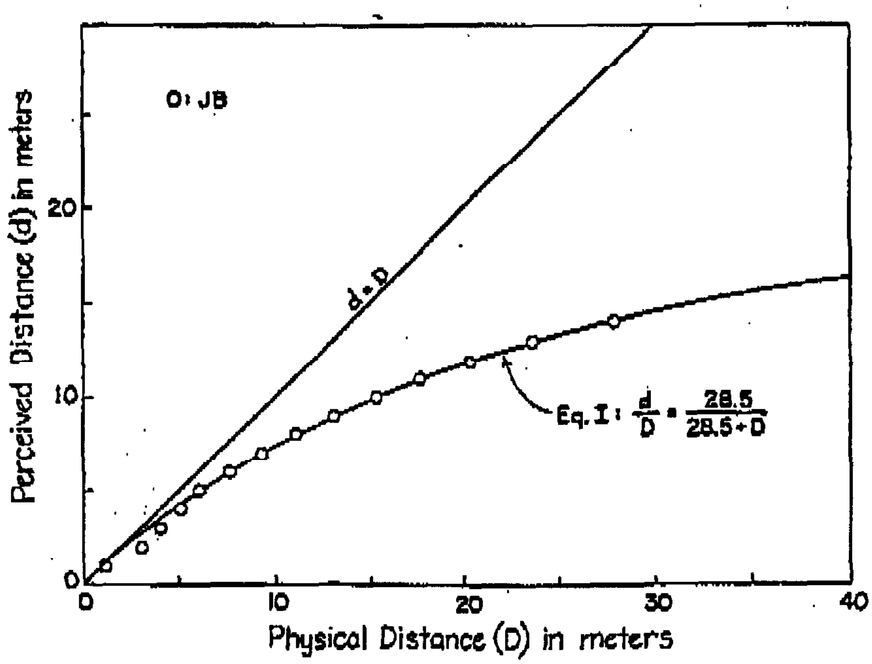

FIG. 5. Perceived distance as a function of physical distance (Experiment I). The circles represent successive equal apparent increments of distance from O to 14 subjective meters. The meter was preferred by this O as the more familiar subjective measuring rod. The data (also shown in Table I) lie on the curve of equation (I) with $A = 28.5$ meters (about 94 feet) and agree closely with the results of the other O of Experiment I.

range than was previously investigated. It was thought that this method would provide a more logically defensible test of the theoretical function (equation I) and would also allow a direct comparison with the procedure adopted by Stevens and Volkmann (15), and Reese (14) for constructing a subjective magnitude function.

O's task was to bisect each one of fourteen distances, ranging from 8 feet to 200 feet, on a large flat lawn. A line was stretched across the lawn, perpendicular to O who stood a few inches away from the end of the anchored line so that she could see all of the line. Bisections were performed by stopping a pointer, which moved back and forth along the line, at a point which appeared to be halfway between the near end of the line and a marker designating the total distance to be bisected. Four sets of half-distance observations were obtained, the standard distances being presented in serial order, alternately ascending and descending. Each bisection was marked with a golf tee, invisible to the observer, and the judged half-distances were measured and the tees removed between series. Two naive Os took part in the experiment.

TABLE I

OBSERVED AND THEORETICAL VALUES OF PHYSICAL DISTANCE AND THE CALCULATED VALUE OF A FOR EACH OBSERVATION OF PERCEIVED DISTANCE

(Data from O: J. B., Exp. I)

Perceived d (meters)	Observed Physical D (meters)	Calculated $A = Dd/(D-d)$	Calculated Eq. I, with $A = 28.5$
		A (meters)	Calc. D (meters)
1	1.08	14	1.04
2	3.09	5	2.15
3	4.12	11	3.35
4	5.12	18	4.65
5	6.10	28	6.05
6	7.59	29	7.60
7	9.39	27	9.28
8	11.08	29	11.10
9	13.10	29	13.14
10	15.34	29	15.40
11	17.60	29	17.91
12	20.37	29	20.74
13	23.56	29	23.94
14	27.96	28	27.52

Weighted Mean $A = 28.5$

TABLE II

OBSERVED AND THEORETICAL VALUES OF HALF DISTANCE (D_1) FOR EACH STANDARD (D_2) FOR THE TWO Os OF EXPERIMENT II

(Calculations from equation (2) using the respective weighted mean values of A)

Observer BS			Observer DM	
D_2 (feet)	D_1 (feet)	Calc. D_1 (feet) ($A = 180$)	D_1 (feet)	Calc. D_1 (feet) ($A = 200$)
8	4.00	3.91	3.67	3.92
10	5.15	4.86	4.58	4.88
12	6.63	5.81	5.00	5.82
16	7.89	7.67	7.83	7.69
20	10.06	9.46	8.50	9.54
30	13.67	13.87	11.67	13.93
40	17.25	18.00	14.83	18.18
60	21.59	25.74	19.58	26.05
80	27.33	32.75	29.58	33.33
100	34.71	39.10	35.50	40.00
120	41.15	45.00	48.08	46.15
150	51.98	52.90	54.25	54.50
180	61.52	60.00	66.00	62.05
200	64.54	64.15	70.00	66.67

The results of this procedure (for each O) are shown in Table II, and the half-judgment function (for the combined data) is plotted as illustrated in Fig. 6. From such half-judgment plots, subjective magnitude functions may be constructed according to the somewhat cumbersome procedure described in detail by Reese (14). For the present problem, the identical functions may be obtained by a much simpler method, by utilizing a convenient property of the basic equation (I).

From equation (I), the half-judgment function must satisfy the relation

$$D_1 = \frac{(\frac{1}{2} D_2) A}{A + (\frac{1}{2} D_2)}. \qquad (2)$$

Equation (I) may also be written in the form

$$d = \frac{D A}{A + D}. \qquad (3)$$

These two functions, equations (2) and (3), are obviously identical. The relation of D_1 to $\frac{1}{2}D_2$ is identical with the relation of d to D. The first is the half-magnitude function, and the second is the subjective magnitude function. This simple relation may be used to obtain a d curve from a D_1 curve or a D_1 curve from a d curve. By simply using a double horizontal scale, a single plotted curve thus serves a dual purpose.

Figure 7 graphically shows this convenient identifying relation between the subjective scale of distance d and the half-judgment curve D_1 replotted from Fig. 6. D_1 is the same curve as d but with the horizontal scale doubled. In Fig. 7, locate the ordinate $d = D_1 = 22$ feet. The distance $D_1 = 22$ from the D_1 curve was judged to be half of $D_2 = 50$ feet. Halving this abscissa yields $D = 25$ feet and, consequently, for $d = 22$, $D = 25$. Accordingly, the experimental curve of Fig. 6 becomes directly a subjective magnitude function by simply using the upper horizontal scale D instead of the lower D_2 scale, and by reading the ordinate as d in place of D_1.

The points marked by triangles on the subjective magnitude function of Fig. 7 are determined by the conventional method (14) by arbitrarily assigning one subjective foot to one physical foot and then proceeding by interpolation of the half-judgment function to discover the successive physical magnitudes to which the suc-

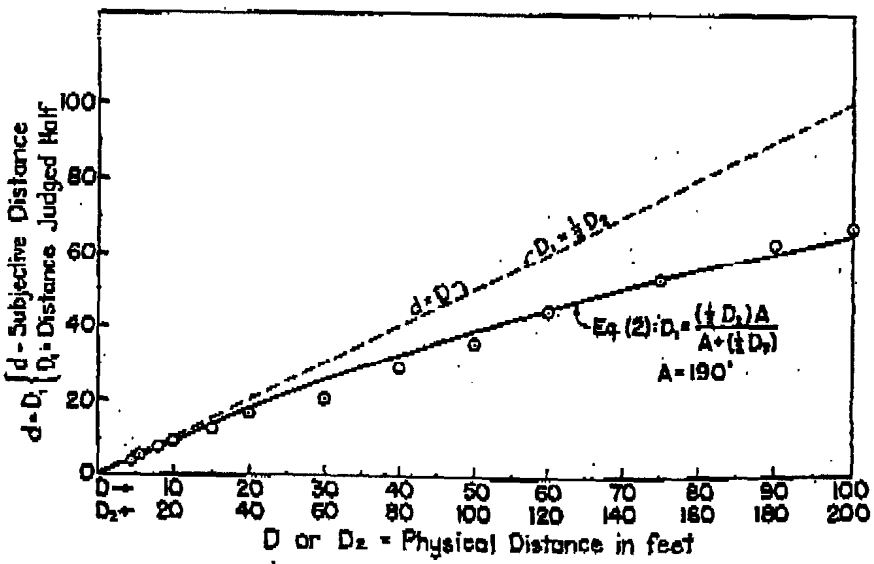

FIG. 6. The mean bisection data of Experiment II are plotted on the curve of equation (2) in the text showing the half-judgment function, or, using the upper horizontal scale (D) and reading d in place of D_1 on the vertical scale, the subjective scale of visual distance. The inclined dashed line shows the objective half distance and objective full distance for the lower and upper horizontal scales, respectively. See text and Fig. 7 for explanation.

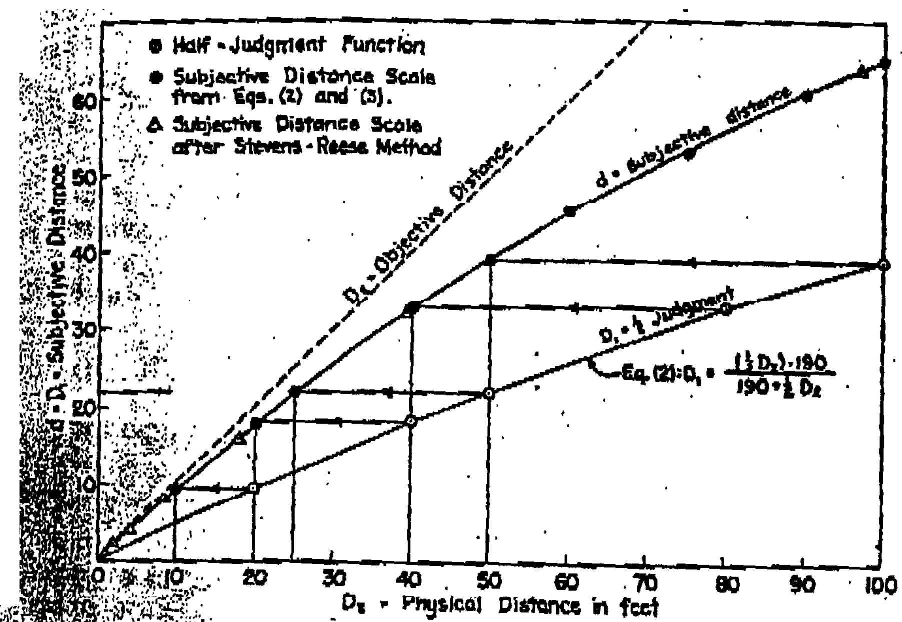

FIG. 7. The magnitude function for subjective distance erected from the half-judgment function by two different procedures. The construction lines show how the magnitude function is obtained from the curve fitted to the mean bisection data by simply halving the horizontal scale, thus utilizing a convenient property of the present formulation. The identical function is obtained by the conventional method (14); see triangles.

cessively doubled numerals (2, 4, 8, 16, 32, 64) should be assigned. This method yields only seven points from these data, whereas the present more expeditious and direct method allows us to use all of the observed points and hence produces a more reliable curve. The function obtained by both methods is identical. The validity of the basic equation (I) is thus doubly confirmed.

SUMMARY

A general unifying law of visual space perception is shown to be capable of rational derivation and quantitative formulation. The derived basic formulas for perceived size and distance as functions of true size and distance are tested and confirmed by application to recorded data of antecedent experiments and to new observations of perceived size and distance. The excellent agreement between observed and calculated values indicates that these formulas are valid not only for the primary case of pure space perception, but also for the more general case of visual space perception (binocular or monocular) with any enrichment or diminution of visual and related cues.

The concept of a maximum limit of perceived distance yields a single parameter, A, in the basic formulas whose magnitude governs the intermediate position of the functions between the boundary conditions expressed by size constancy and retinal image, respectively. The numerical value of this parameter, A, for a given observer is shown to depend upon the available perceptual cues to distance, and, hence, provides a convenient direct index or measure of "phenomenal regression."

The next indicated study is a planned series of experiments to determine the conditions and laws of variation of the parameter A. The work-

ing definitions and formulas developed here, and the data thus far analyzed, offer a starting point and a guide for further research in space perception.

APPENDIX I

Derivation of Formulas from Luneburg's Basic Metric of Visual Space

The basic metric of visual space, as formulated and established by Luneburg (12, Eq. 3.781), is

$$ds^2 = \overline{d\,(d)}^2 + M^2\,(d\varphi^2 + \cos^2\varphi \cdot d\theta^2), \quad (1)$$

where

d = visual (perceived) distance,
s = visual (perceived) size,
φ = horizontal angle (azimuth),
θ = vertical angle (altitude),
M = linear size factor = $ds/d\varphi$.

Luneburg makes this basic metric his point of departure for transformations into elliptic, Euclidean, and hyperbolic space geometries, respectively. With the same point of departure, we shall make our Euclidean transformation with the following consistent substitutions:

$M = d$. (Hence $ds = d \cdot d\varphi$.)

True (objective) distance = D = a/γ, where

a = interpupillary distance,
γ = angle of binocular convergence.

Perceived distance = $d = \dfrac{a}{\gamma + \mu}$.

Hence

$$d\,(d) = -\frac{a \cdot d\gamma}{(\gamma + \mu)^2}.$$

(μ (mu) = a constant added to γ. It limits maximum perceived d, keeping d finite when γ is zero or imperceptibly small. In other words, μ = the minimum or threshold value of *perceived* convergence or horizontal disparity in viewing very distant objects.)

Substituting M and d (d) in the basic metric, equation (1),

$$ds^2 = \frac{a^2}{(\gamma + \mu)^2} \cdot \left(\frac{d\gamma^2}{(\gamma + \mu)^2} + d\varphi^2 + \cos^2\varphi \cdot d\theta^2 \right). \quad (2)$$

Note the form of the term $\dfrac{d\gamma}{(\gamma + \mu)}$. This is a logarithmic form, corresponding to the Weber-Fechner law. Sensitiveness to $d\gamma$ diminishes with increasing γ. Also note: Vision is $1/(\gamma + \mu)$ times as sensitive to $d\gamma$ as to $d\ \varphi$ or $d\ \theta$. Thus the visual distance factor is $1/(\gamma + \mu)$ times the visual size factor.

The transformed metric, equation (2), contains two constants (a, μ) varying with the observer.

By equation (2), with $d\ \varphi = 0$ and $d\ \theta = 0$,

$$d = a/(\gamma + \mu).$$

But

$$D = a/\gamma.$$

Hence,

$$\frac{d}{D} = \frac{\gamma}{(\gamma+\mu)} = \frac{a/\mu}{a/\mu + a/\gamma} = \frac{A}{A+D},$$

with

$$A = a/\mu.$$

(Two observer constants are thus replaced by one.)

Similarly, by equation (2) with $d\gamma = 0$ and $d\theta = 0$,

$$ds = M \cdot d\varphi = d \cdot d\varphi, \text{ or } s = d \cdot \varphi.$$

But objective true size S_0 (to the same scale as D) is given by

$$dS_0 = D \cdot d\varphi, \text{ or } S_0 = D \cdot \varphi.$$

Hence

$$\frac{s}{S_0} = \frac{d}{D} = \frac{A}{A + D}. \quad (3)$$

Let σ (sigma) $= A/B$, where

$$B = A + \delta,$$

and δ (delta) $= D$ at normal viewing distance. Then, by definition, subjective true size S is given by equation (3) as

$$\frac{S}{S_0} = \frac{A}{A+\delta} = \frac{A}{B} = \sigma. \qquad (4)$$

From equations (3) and (4),

$$\frac{s}{S} = \frac{B}{A}\cdot\frac{d}{D} = \frac{1}{\sigma}\cdot\frac{d}{D} = \frac{B}{A+D}. \qquad (5)$$

Note: d is perceived distance, to the same scale as visual size s. ($\varphi = s/d$.)

D is measured (objective) distance, to the same scale as measured (objective) size S_0. ($\varphi = S_0/D$.)

A, B, D, and d are given in the same units.

S_0, S, and s are given in the same units.

We have thus derived two simple formulas, for perceived distance and perceived size, respectively:

$$\frac{d}{D} = \frac{A}{A+D}, \qquad (I)$$

$$\frac{s}{S} = \frac{B}{A+D}. \qquad (II)$$

APPENDIX II

Derivation of Formulas from Principles of Perspective

The basic geometry of vision is represented in Fig. A1, and the basic geometry of perspective is shown in Fig. A2.

Referring to Fig. A2, the compression of distance in perspective corresponds to the compression of perceived *distance* in visual space; and the perspective reduction of size (width or height) between parallel lines receding from the observer corresponds to the reduction of perceived *size* in visual space. The parallel horizontal lines receding from the observer are seen in perspective as converging to a "vanishing point" in the horizon line. These proportional relations common to perspective and visual space are shown in Fig. A2. (VP is the "vanishing point"; DP is the "division point." From the objective plan below the ground line, the geometrical perspective is constructed above the ground line as shown.)

In Figs. A1 and A2,

- D = True (objective) distance; d = Perceived distance.
- S_0 = True (objective) size; s = Perceived size.
- A = Horizon distance = Apparent distance of vanishing points in the horizon.

By Fig. A1, the visual angle φ at the nodal point of the eye is given by

$$\varphi = \frac{s}{d} = \frac{S_0}{D}.$$

Hence,

$$\frac{s}{S_0} = \frac{d}{D}. \qquad (1)$$

By similar triangles in Fig. A2,

$$\frac{s}{S_0} = \frac{A-d}{A}. \qquad (2)$$

From equations (1) and (2),

$$\frac{d}{D} = \frac{A-d}{A}. \qquad (3)$$

This reduces to

$$\frac{d}{D} = \frac{A}{A+D}. \qquad (I)$$

Hence, by equation (1),

$$\frac{s}{S_0} = \frac{A}{A+D}. \qquad (4)$$

By definition of the proportionality constant σ (sigma),

$$\sigma = \frac{S}{S_0} = \frac{A}{B}. \qquad (5)$$

From equations (4) and (5),

$$\frac{s}{S} = \frac{B}{A + D}. \qquad (II)$$

The two basic formulas (I) and (II), thus written directly from the geometry of perspective, are identical with the two basic formulas derived rigorously in Appendix I from Luneburg's more theoretical metric of visual space.

APPENDIX III

Derivation of Basic Formula by Composition of the Laws of Size Constancy and Visual Angle

It is known that the law of perceived size is intermediate between the law of retinal image (visual angle) and the law of size constancy (2, 9). This intermediate relationship has been called "regression to the real object" (16).

The law of retinal image is expressed by

$$\frac{s}{S} = \frac{\delta}{D}, \qquad (1)$$

where δ (delta) is the distance at which an object appears "true size" ($s = S$). This would make the perceived size s of an object vary inversely with its distance D.

The law of size constancy is expressed by

$$\frac{s}{S} = \frac{A}{A}. \qquad (2)$$

This would make $s = S$, independent of distance.

By composition of equations (1) and (2), combining the two extreme laws into a single generalized intermediate law, we have

$$\frac{s}{S} = \frac{A + \delta}{A + D}. \qquad (II)$$

The value of A (or the ratio of $A:D$) determines the relative position of the interpolated law (II) between the two extremes, (1) and (2). Compare the Thouless (16) coefficient. For $A = 0$, equation (II) becomes the law of retinal image. For A = infinity, equation (II) becomes the law of size constancy. For $A = D$(or $D = A$), equation (II) gives a value exactly midway between the two limiting laws, (1) and (2).

The magnitude of A (between zero and infinity) is thus a measure of the

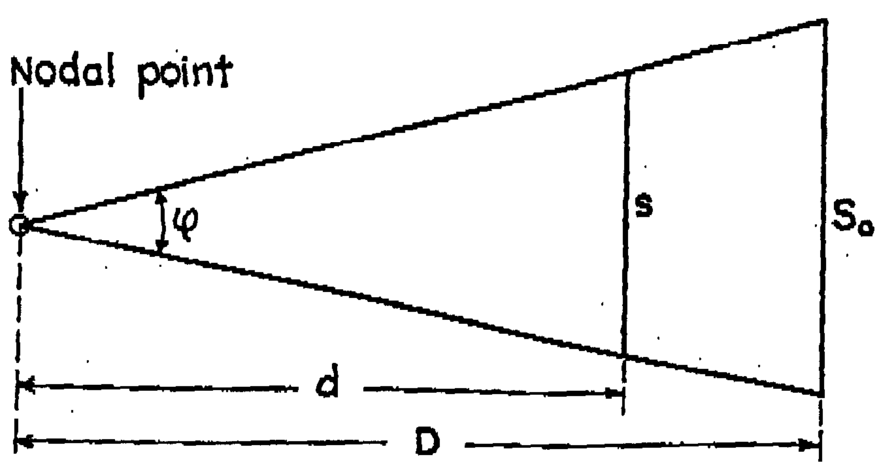

FIG. A1. Geometry of visual space.

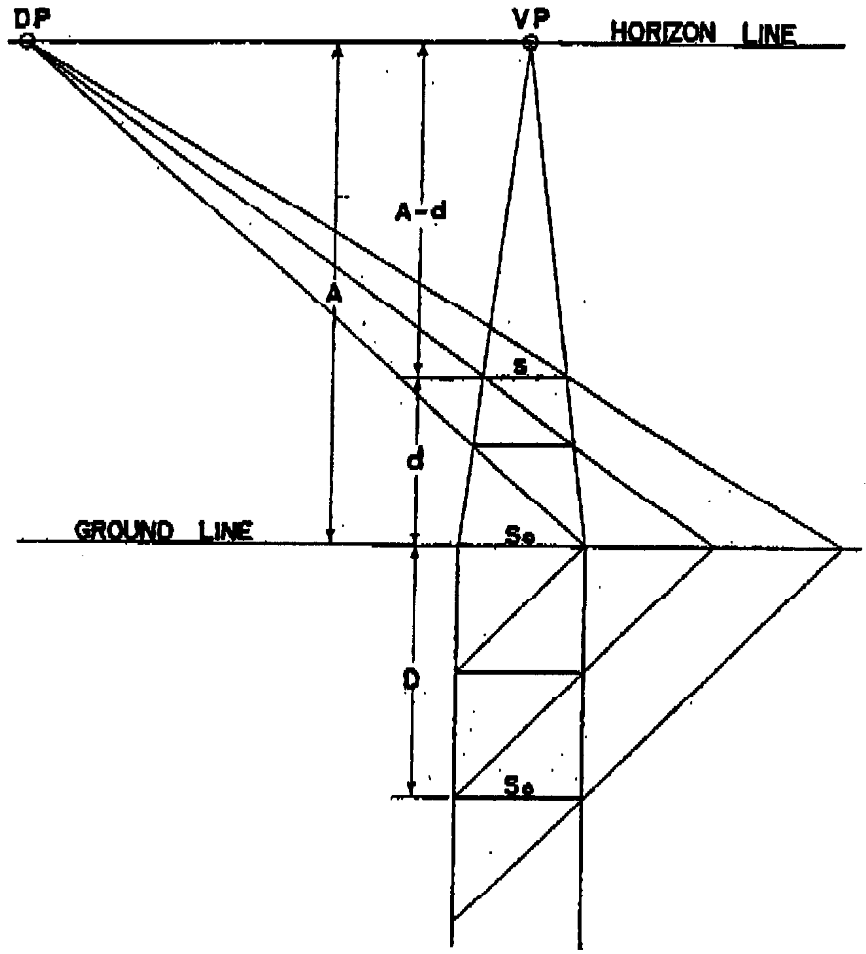

FIG. A2. Perspective geometry.

"regression to the real object." For $A = 0$, the regression is zero. For $A =$ infinity, the regression is complete. For $A = D$, the departure is half-way. The difference between retinal image and size constancy is divided into two parts by the perceived size; the ratio of the two parts is $A:D$.

At small values of D (compared to A), the perceived size, by equation (II), is close to size constancy; the difference is so small as to be missed by investigators.

At very large values of D (compared to A), the perceived size, by equation (II), is close to the retinal image law. The sun and the moon appear nearly the same size because their retinal images are nearly the same size, although their real sizes are 880,000 miles and 2160 miles, respectively, a ratio of 407 to 1.

Equation (II) is the basic formula for visually perceived size, identical with the basic formula derived in the preceding Appendices. It is a generalized, unifying expression, embracing the laws of size constancy and retinal image as special or limiting cases and bridging the gap between those two extreme laws.

APPENDIX IV

Experimental Determination of the Visual Constant A

The visual constant A for a given O under given conditions may be determined experimentally in a variety of ways, either from a single size or distance observation of an object of known physical size and distance, or from two or more related size or distance observations. A few of the convenient, practical methods are outlined below:

1. *From a size observation of an astronomical object.* The size of retinal image or the subtended visual angle φ (phi) for any object is given by

$$\varphi = \frac{S_0}{D} = \frac{s}{d}. \qquad (1)$$

For an astronomical object, the physical distance D is enormous or practically infinite relative to A. For this condition, the basic distance formula (I) yields $d = A$. Hence

$$A = d = \frac{s}{\varphi}. \qquad (2)$$

Thus A is given directly if S_0 and D are known and s is observed.

Thus, for the sun,

$$\varphi = \frac{S_0}{D} = \frac{880{,}000 \text{ miles}}{92{,}000{,}000 \text{ miles}} = 0.0096 \text{ radians.}$$

If the apparent size of the sun is

$$s = 15 \text{ in.} = 1.25 \text{ ft.},$$

then

$$A = d = \frac{s}{\varphi} = \frac{1.25 \text{ ft.}}{0.0096} = 130 \textit{ ft.}$$

Similarly, for the moon,

$$\varphi = \frac{S_0}{D} = \frac{2160 \text{ miles}}{244{,}000 \text{ miles}} = 0.0090 \text{ radians.}$$

If the apparent size of the moon is

$$s = 14 \text{ in.} = 1.17 \text{ ft.},$$

then

$$A = d = \frac{s}{\varphi} = \frac{1.17 \text{ ft.}}{0.0090} = 130 \textit{ ft.}$$

Note that the sun and the moon subtend nearly equal visual angles φ, that of the sun being somewhat larger. Because both objects are perceived at the same distance ($d = A$), the apparent sizes are proportional to the respective visual angles. At astronomical distances, visual size more nearly follows the law of the retinal image.

2. *From a size observation of a non-astronomical object.* By the basic formula (II) for visual size,

$$r = \frac{s}{S_0} = \frac{A}{A + D}, \qquad (3)$$

where r is the reduction ratio for visual size s from physical size S_0. Solving equation (3) for A,

$$A = \frac{r}{(1 - r)} \cdot D. \qquad (4)$$

This gives A directly from a single observation when physical distance D, physical size S_0, and perceived size s are known for any given object.

3. *From perceived distance of a non-astronomical object.* The basic formula for visual distance is

$$\frac{d}{D} = \frac{A}{A + D}. \qquad (I)$$

Solving for A,

$$A = \frac{D \cdot d}{D - d}. \qquad (5)$$

This gives A directly from a single distance-observation for any object, when physical distance D and visual distance d are given.

4. *From two size-observations of the same object.* Let s_1 be the perceived size of a given object at measured distance D_1, and s_2 the perceived size of the same object at measured distance D_2. By the basic formula (II) for visual size,

$$\frac{s_1}{S} = \frac{B}{A + D_1} \qquad (6a)$$

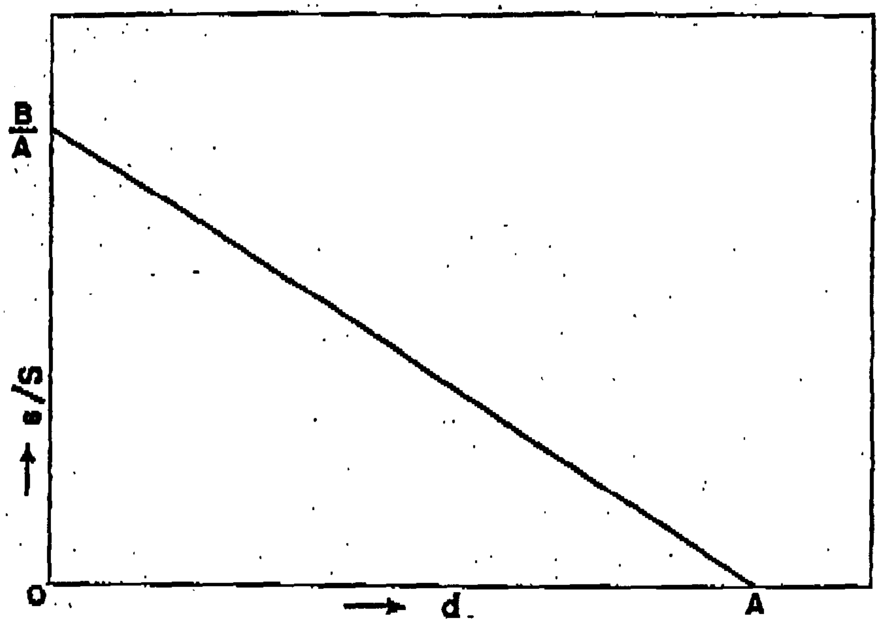

FIG. A3. Perceived size and perceived distance.

and

$$\frac{s_2}{S} = \frac{B}{A + D_2} \qquad (6b)$$

From equations (6a) and (6b),

$$\frac{s_1}{s_2} = \frac{A + D_2}{A + D_1}. \qquad (7)$$

Solving equation (7) for A,

$$A = \frac{D_2 s_2 - D_1 s_1}{s_1 - s_2}. \qquad (8)$$

This gives A directly when the two physical distances, D_1 and D_2, and the two respective perceived sizes, s_1 and s_2, of a given object are known.

5. *From a half-distance observation.* Let D_1 be the measured distance of the apparent (visually perceived) midpoint of the measured distance D_2. By the basic formula (I), the half-distance observation is expressed by the pair of equations:

$$d_1 = \frac{A \cdot D_1}{A + D_1} \qquad (9a)$$

and

$$d_2 = 2d_1 = \frac{A \cdot D_2}{A + D_2}. \qquad (9b)$$

Solving this pair of equations for A

$$A = \frac{D_1 \cdot D_2}{D_2 - 2D_1}. \qquad (10)$$

This gives A directly when any measured distance D_2 and the apparent half-distance D_1 are given.

6. *From a series of observations in a visual size or distance experiment.* As a rule, a larger number of observations make up a size or distance experiment. In such case, the best mean value of A is obtained as a weighted mean of the values of A computed for the individual observations (using the respective distances D as the weight factors). Other convenient methods of curve-fitting or short-cut applications of the Theory of Least Squares may be used to determine the best mean value of A to fit the series of observations.

After A is determined, B is given by $B = A + \delta$, where δ (delta) is the normal viewing distance for the observer, usually 1 or 2 ft.

7. *From coupled observations of perceived size and perceived distance.*

From two or more observations of a given object viewed at different distances, the recorded values of perceived size may be plotted directly against the corresponding values of perceived distance. The resulting graph (Fig. A3) is a straight line, inclined downward toward the right. The relation is given by

$$\frac{s}{S} = \frac{B}{A} \cdot \frac{A - d}{A}. \qquad (11)$$

For $d = 0$, $s/S = B/A$. For $s/S = 0$, $d = A$. Accordingly the straight line graph intercepts the vertical axis at B/A and the horizontal axis at A. The two parameters, A and B, are thus given directly by the plotted graph.

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